FEM Quasi-3D Modelling of Responses to Artificial Recharge in a Multiaquifer System

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Abstract : Quasi three-dimensional (quasi-3D) modelling is playing a major role in analysis of groundwater flow in a multiaquifer system today and will probably for a long time in future. It also can be an efficient tool for artificial recharge modelling. Because the role of artificial recharge as a tool for environmental management increases more and more, modelling of responses to artificial recharge in a multiaquifer system has a practical significance. This paper deals with some practical aspects of the development of a general FEM quasi-3D model based on an algorithm employing the combined techniques of FEM and convolution integral. The results of analysis of responses to well recharging in a multiaquifer system will be presented for a case study of Bangkok city where land subsidence due to groundwater extraction is a serious environmental issue to be resolved, and for which artificial recharge has been recommended as a supplemental means of mitigation.

1. INTRODUCTION

Practical analyses of groundwater flow in a multiaquifer system which have been done mostly based on the quasi-3D models which have advantages over a fully 3D groundwater model in terms of input of hydraulic properties of aquifers and aquitards as well as the computation efforts. The quasi-3D model can be safely applied if the permeability of the aquifers is at least two orders of magnitude larger than that of the aquitards and this happens in most practical situations. The term of *the quasi three-dimensional model* was introduced by Bredehoeft and Pinder (1970). Its origin, however, derived from a much older theory of leaky aquifer which deals with analytical solutions of aquifer-aquitard interactions. A quasi-3D model considers only the horizontal flow in aquifers and the vertical flow in the adjacent aquitards. The assumption that the horizontal velocity components change little with the height of the aquifer is very significant, turning the flow problem in a multiple aquifer system from *the hydrodynamic theory* to *the hydraulic theory* (De Wiest, 1961). The development of the numerical quasi-3D models has evolved around some main points such as : (i) how to solve numerically the 2D horizontal flow equation in the aquifer under different boundary conditions, either by the Finite Difference or Finite Element method ; (ii) how to solve 1D vertical flow equations in aquitards, analytically or numerically, taking into account nonlinearity of aquitard properties or not; (iii) how to couple these equations via the leakage flux. Two groups of quasi-3D models have been developed based on two different ways of coupling, i.e. fully numerical and semi-analytical. The semi-analytical approach solution is mathematically neat and efficient in saving computation time. However, the iteration process of computing the power series when determining convolution integral can create truncation error and requires a lot of storage. The fully numerical procedure may be slower in computation time, but it may not require an iteration process and can account easily for an arbitrary geometry and non-linear property of aquitards. More details on development of the quasi-3D models can be found in Herrera et al. (1980); Neuman et al. (1982); Giao (1997).

Groundwater extraction at a rate of more than one million cubic meter per day from the underneath aquifer system has caused very large drawdowns, inducing land subsidence of Bangkok city in a large area. Because the present water supply can not meet the increasing water demand brought about by rapid economic growth of the city the extraction of groundwater for municipal and industrial uses will continue for years ahead. Thus, besides the control of groundwater pumping the other measures including artificial recharge have to be considered. To develop artificial recharge as a supplemental means of groundwater management, modelling of responses of the aquifer system to artificial recharge is much needed. Responses to artificial recharge in a multiaquifer system are understood here as the hydraulic responses of the recharged aquifers to well recharging and compression or recompression of the adjacent aquitards. As the Bangkok aquifer system consists of nine confined aquifers extending to 500 m deep, interactions between its aquifers and aquitards have to be addressed in any artificial recharge modelling. A quasi-3D model can be a very suitable tool for such hydrogeological situation.

2. DEVELOPMENT OF THE QUASI-3D MODEL

A quasi-3D model was developed based on the combined techniques of FEM and convolution integral. The main steps of development can be summarized as follows : (i) formulate governing equations of a Quasi-3D model unit; (ii) analytically solving 1D vertical flow equations based on methods of separation of variables and Fourier series to find out head distribution in the aquitards; (iii) determine leakage fluxes from adjacent aquifers and aquitards into the computed aquifer by means of the technique of convolution integral (Huyakorn and Pinder, 1983) and incorporate them into the 2D horizontal flow equation; (iv) FE formulation of the 2D horizontal flow equation to obtain linear equations which can be coded in a program for artificial recharge modelling.

The governing equations of the quasi-3D model which consist of a 2D horizontal flow equation in the aquifer and

1D vertical flow equations in the adjacent aquitards for a typical unit of the quasi-3D (Figure 1) are as follows :

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h_i}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_y \frac{\partial h_i}{\partial y}\right)+L+Q=S_i \frac{\partial h_i}{\partial t} \tag{1a}$$

$$\frac{\partial}{\partial z}\left(k_j \frac{\partial h_j}{\partial z}\right)=s_{s,j} \frac{\partial h_i}{\partial t} \tag{1b}$$

$$\frac{\partial}{\partial z}\left(k_{j+1} \frac{\partial h_{j+1}}{\partial z}\right)=s_{s,j+1} \frac{\partial h_{i+1}}{\partial t} \tag{1c}$$

Where : h_i is the head in the computed aquifer; T_x, T_y are the aquifer transmissivities along x & y-directions; S_i is the aquifer storage; Q is a sink/source term; L is the total leakage flux, i.e. $L = q_j + q_{j+1}$ with q_j, q_{j+1} are fluxes from the overlying (j^{th}) and underlying ($j+1^{th}$) aquitards to the aquifer; h_j & h_{j+1} , k_j & k_{j+1} and s_j & s_{j+1} are heads, permeabilities and specific storages of the j^{th} and $j+1^{th}$ aquitards, respectively.

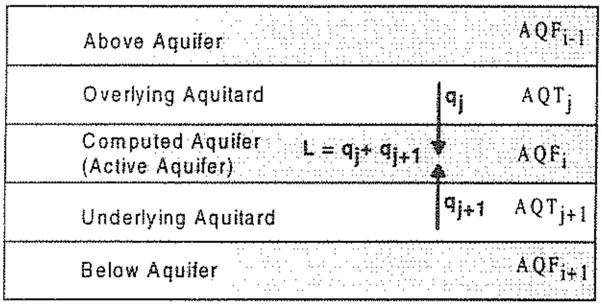

Figure 1 A Typical Unit of the Quasi-3D Model

The areal domains of the aquifers in the multiaquifer system to be modelled are discretized in form of superimposed meshes having the same number of nodes and elements (Figure 2). By applying the Galerkin's procedure, *the weak statement* of the quasi-3D model can be written for a generic domain, Ω^e , as below :

$$\int_{\Omega^e}\left(T_x \frac{\partial N_k}{\partial x}\frac{\partial N_l}{\partial x}+T_y \frac{\partial N_k}{\partial y}\frac{\partial N_l}{\partial y}\right)h_k d\Omega^e +$$
$$\int_{\Omega_e} S N_k N_l\left(\frac{\partial h_k}{\partial t}\right)h_k d\Omega^e - \int_{\Omega_e} Q N_l d\Omega^e -$$
$$\int_{R^e} N_l\left(T_x \frac{\partial N_k}{\partial x}+T_y \frac{\partial N_k}{\partial y}\right)h_k dR^e - \int_{\Omega_e} L N_l d\Omega^e = 0 \tag{2}$$

Where : Indexes k, l are node numbers. N_k, N_l are linear shape functions for a quadrilateral element in the local coordinates (Smith and Griffiths, 1982). The leakage flux was found as :

$$L=\left[\frac{k_j}{b_j}h_i^{m+1}(1+\gamma)-\frac{k_j}{b_j}(h_{i-1}^m+\gamma h_i^m)+\frac{k_j}{b_j}\beta\right]-$$
$$\left[\frac{k_{j+1}}{b_{j+1}}h_i^{m+1}(1+\gamma')-\frac{k_{j+1}}{b_{j+1}}(h_{i+1}^m+\gamma' h_i^m)+\frac{k_{j+1}}{b_{j+1}}\beta'\right] \tag{3}$$

Where m and (m+1) are indexes for time steps, Δt; (i-1), i, (i+1) are indexes for aquifers; j, (j+1) are indexes for aquitards (see Figure 1); b_j , b_{j+1} are the thickness of the overlying and underlying aquitards. The parameters $\alpha_{n,j}$, $\alpha_{n,j+1}$, β, β', γ, γ' are determined by the following relationships :

$$\alpha_{n,j}=\frac{n^2\pi^2 k_j}{b_j^2 S_{s,j}} \qquad \alpha_{n,j+1}=\frac{n^2\pi^2 k_{j+1}}{b_{j+1}^2 S_{s,j+1}}$$

$$\beta=2\sum_{n=1}^{\infty} e^{-\alpha_{n,j}\Delta t_{m+1}} I_{n,j}^m \qquad \beta'=2\sum_{n=1}^{\infty} e^{-\alpha_{n,j+1}\Delta t_{m+1}} I_{n,j+1}^m$$

$$\gamma=2\sum_{n=1}^{\infty}\frac{1-e^{-\alpha_{n,j}\Delta t_{m+1}}}{\alpha_{n,j}\Delta t_{m+1}} \qquad \gamma'=2\sum_{n=1}^{\infty}\cos n\pi\frac{1-e^{-\alpha_{n,j+1}\Delta t_{m+1}}}{\alpha_{n,j+1}\Delta t_{m+1}}$$

... (4)

And the convolution integral can be computed by the following recurrence formula :

$$I_{n,j}^{m+1}=e^{-\alpha_{n,j}\Delta t_{m+1}} I_{n,j}^m+\frac{1}{\alpha_{n,j}}\left(\frac{\Delta h}{\Delta t}\right)_{m+1}(1-e^{-\alpha_{n,j}\Delta t_{m+1}})$$

...(5)

By introducing the leakage flux, L, from Eq. 3 in to Eq. 2 and applying further treatments on boundary conditions, sink/source and time approximation, Eq. 2 can be recast in the form of a matrix and vector equation.

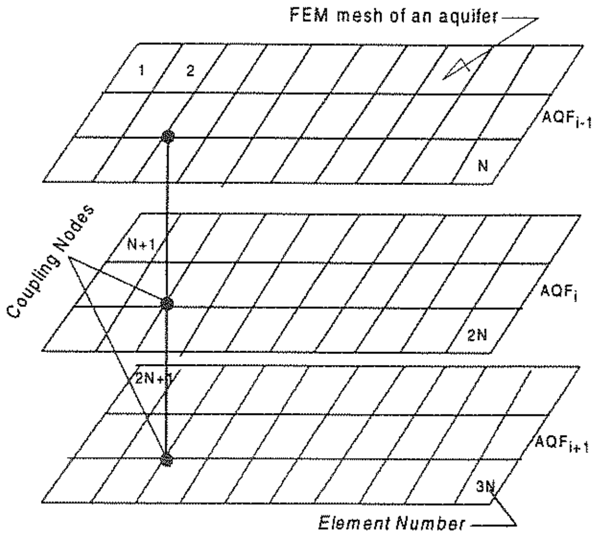

Figure 2 Mesh of a FEM Quasi-3D Model

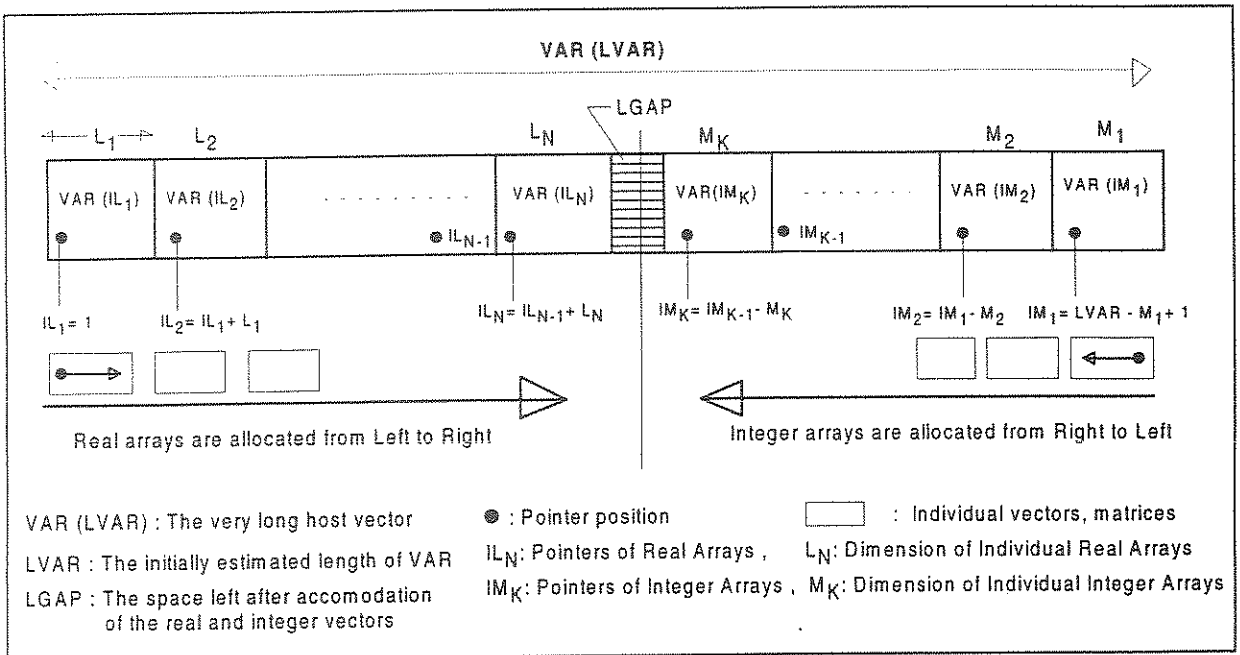

Figure 3 Pseudo-dynamic Allocation Storage in QSI3D

The subsequent assembling of Eq. 2 leads to the global matrix equations based on which a FEM program can be developed. Such a program, named QSI3D, was written in FORTRAN 77 for groundwater and artificial recharge modelling (Giao, 1997). Some notable features of QSI3D are as follows : (i) incore memory management by pseudodynamic allocation; (ii) ability to work with both areal and radial quasi-3D models for recharge modelling; (iii) ability to cope with the time-dependent change of permeability due to clogging; (iv) to analyze groundwater flow schemes, taking into account both leakage and storage of aquitards. Nowadays, with the fast progress of PC hardware, especially in term of RAM and CPU, as well as the availability of the object-oriented languages such as C^{++}, Visual C^{++}, Visual Basic etc. a quasi-3D model can be tailor-developed as a user-friendly PC-based program without many difficulties. As FORTRAN still remains a widely used language, it is the authors' opinion that the following aspects are useful for practical development of a quasi-3D model for recharge modelling in a multiaquifer system, namely :

(i) Incore memory management by pseudo-dynamic allocation : It is known that for most practical FEM programs the ratio between the amount of internally generated data (usually related to the stiffness and global matrices) and the amount of externally available data (usually related to mesh geometry, material properties etc.) is considerably high. The usual way of storing internal data structure by defining their types and sizes through DIMENSION statements at the beginning of a program can lead to underestimation of the memory required for some arrays and to overestimation for some others, as well as frequent need of recompilations of programs to adjust array dimensions for different applications. To overcome this disadvantage, a technique called "*pseudodynamic storage allocation*" is often employed. Pseudodynamic allocation places, one by one, all individual arrays, either matrices or vectors, into a single and very long vector according to the precalculated pointers. It has many advantages, i.e., optimization of memory requirements of program, reduction of fragmentation of the data structure which is now compacted in a single array, it has positive effects on compilers since it requires fewer internal tables an resources. In the QSI3D program, the technique of pseudodynamic storage allocation was also employed. Most arrays are placed into a real vector named VAR of single precision, which is declared in the main program by a COMMON statement. First, all the pointers are calculated to help linking the vector VAR to different arrays in the subroutines through argument lists. VAR can accommodate the real arrays of both single and double precision as well as the integer arrays. The real arrays start being placed from left end to right, whereas the integer arrays from right end to left. Implementation of pseudodynamic allocation by reserving storage in a single vector, VAR, is illustrated in Figure 3.

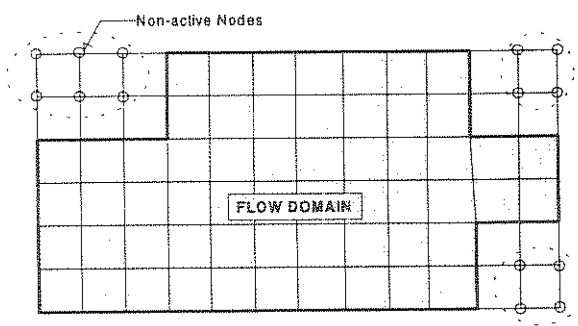

Figure 4 Treatment of non-active nodes

(ii) Treatment of non-active nodes : The use of quadrilateral elements in discretization of flow domain has an advantage in simple mesh generation, but drawback in modelling flow domain of irregular boundaries. To overcome this, non-active nodes are introduced (Figure 4). The non-active nodes covers areas lying between the flow domain boundaries and the mesh boundaries, they are not involved, however, in processes of forming element and global matrices. There are needed three subroutines to treat non-active nodes, i.e., (i) the first to read the non-active nodes and does renumbering of degrees of freedom (D.O.F) of the whole mesh; (ii) the second to do adjustment of numbering for special nodes such as sinks, sources or those on boundaries; (iii) the third to do readjustment of D.O.F numbering for output.

(iii) Time-dependent change of the hydraulic conductivity near the well : Once recharge operation commences the clogging starts to develop, reducing gradually the hydraulic conductivity of the medium around the well. Consequently, the head inside the recharge well is continuously increased, affecting the operation of the recharge well. This nonlinearity should be treated in modelling of responses to recharge. A new index, Recharge Clogging Factor (RCF), defined as the ratio between the hydraulic conductivity of the zone near the well during recharge and discharge periods, i.e., k_r and k_d, respectively, can be used to account for this nonlinearity. An empirical rule of RCF change with the pumping time can be established by a procedure based on the field test data and put in the program at the part where the element stiffness matrix is formed (Giao, 1997). For example, in the case of recharging into the Upper Bangkok, the following rule was determined : $RCF = k_r/k_d = 0.98\ t^{-0.11}$, where t is the pumping time, k_d is the hydraulic conductivity determined from a pumping test carried out before the recharging period.

3. MODELLING OF RESPONSES TO ARTIFICIAL RECHARGE IN THE BANGKOK AQUIFER SYSTEM

Bangkok (Figure 5) is situated on the Chao Phraya river flood plain, whose basement depth varies probably from hundreds to thousands meters. The basement is overlain by a sequence of unconsolidated, deltaic, marine and alluvial sediments which constitute the Bangkok aquifer system.

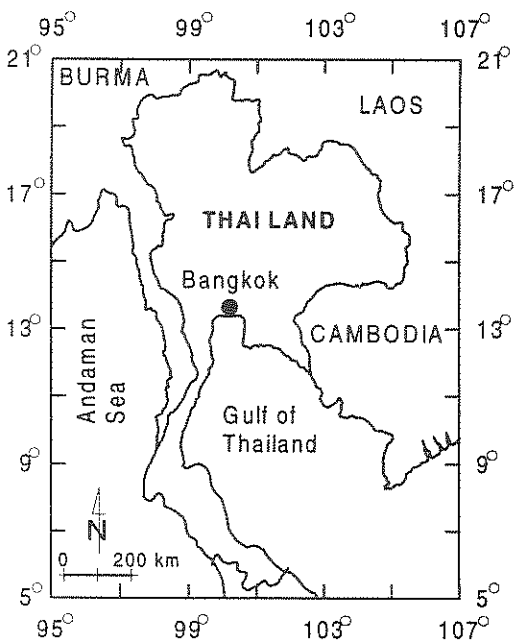

Figure 5 Location of Bangkok, Thailand

Table 1 A sketch of the Bangkok aquifer system

Depth (m)	Hydrogeological Units	Hydraulic Parameters
15	Bangkok CLAY (BC Clay)	*k = 4.06E-5 (m/d), Ss = 6.75E-3 (1/m)*
25	Upper Bangkok Aquifer, UBK	K = 13.7 m/d S = 0.50 E-3
30	Hard CLAY (LBK Clay)	*k = 4.65E-5 (m/d), Ss = 7.70E-4 (1/m)*
60	Lower Bangkok Aquifer , LBK	K = 13.7 m/d S = 0.50 E-3
70	Hard CLAY (PD Clay)	*k = 1 .77E-6 (m/d), Ss = 2.12E-4 (1/m)*
120	Phra Pradaeng Aquifer, PD	K = 17.8 m/d S = 0.50 E-3
130	Hard CLAY (NL Clay)	*k = 9.01E-7 (m/d), Ss = 1.67E-4 (1/m)*
180	Nakhon Luang Aquifer, NL	K = 16.1m/d, S = 0.50 E-3
190	Hard CLAY (NB Clay)	*k = 7.29E-7 (m/d), Ss = 1.15E-4 (1/m)*
240	Nonthaburi Aquifer, NB	K = 17.1m/d, S = 0.50 E-3
250	Hard CLAY (SK Clay)	*k = 3.37E-7(m/d), Ss = 7.72E-4 (1/m)*
300	Samkhok Aquifer, SK	K = 16.1m/d, S = 0.50 E-3
310	Hard CLAY (PT Clay)	*k = 1.4E-7 (m/d), Ss = 1.17E-4 (1/m)*
360	Phayathai Aquifer, PT	K = 10.8 m/d, S = 0.50 E-3
370	Hard CLAY (TB Clay)	*k = 2.0E-7 (m/d), Ss = 0.5E-4 (1/m)*
420	Thonburi Aquifer, TB	K = 3.7m/d, S = 0.50E-3
430	Hard CLAY (PN Clay)	*k = 2.0E-7 (m/d), Ss = 0.5E-4 (1/m)*
500	Pak Nam Aquifer, PN	K = 17.5m/d, S = 0.50 E-3
	Bedrock	

The Bangkok aquifer system has nine aquifers (Table 1). Groundwater extraction for the municipal and industrial uses comes mainly from three aquifers, namely, Phra Pradaeng, Nakhon Luang and Nonthaburi. Extensive pumping, especially in the 1970-1980's, led to significant declines of piezometric heads, maximum up to 40-50 m in each exploited aquifer mentioned above, and, consequently to land subsidence of Bangkok and its vicinity. The maximum subsidence observed during 1933 - 1978 was 85 cm and that between 1978 and 1987 was 75 cm. As a part of an intergrated management of water supply and mitigation of land subsidence for Bangkok city artificial recharge was proposed. In the initial stages of application of artificial recharge a field experiment and modelling of possible responses in the injected medium to recharge are the most needed works. As recharging is costly a pilot test is usually carried out at a shallow depth. A field experiment of well recharging into the uppermost aquifer of the Bangkok aquifer system, which was carried out between 1993 and 1995 by the Asian Institute of Technology, showed that recharging could be successfully done into the Upper Bangkok aquifer, its results also recommended a direct recharging into the other deeper aquifers in the future. Thus, at present, the responses of the deep aquifers to artificial recharge can only be investigated based on a modelling work. The scopes of artificial recharge modelling are aimed at answering the following questions : (i) what would be the changes in the piezometric heads in the aquifers as well as the dissipation of pore pressure in the adjacent aquitards for different patterns of pumping and recharging in term of numbers of recharge well and the injection depths; (ii) what would be the possible changes in the geotechnical parameters of the medium around the recharge wells such as permeability, storage coefficient, vertical and horizontal stresses etc. which, in turn, may affect on efficiency of recharge operation; (iii) what would be the rebounds of the injected soil after it was compressed by a previous pumping; (iv) what would be the soil parameters needed for analysis as well as a proper procedure of analysis. The third question related to the rebounds artificial recharge can bring back is of a particular importance in the application for land subsidence mitigation and it will be presented in details below for a case of study for the Bangkok aquifer system. The steps of artificial recharge modelling can be summarized as follows :

a) Building a hydrogeological model of the Bangkok aquifer system.

b) Assume a pumping pattern and carry out a steady-state analysis of groundwater flow in the Bangkok aquifer system based on the program QSI3D to find out the possible drawdowns in the aquifers. Calculate the instantaneous compression of the aquifers.

c) Input the drawdowns obtained from the groundwater analysis to a consolidation model and calculate the pore pressure dissipation in the adjacent aquitards as well as the corresponding compressions.

d) Assume different schemes of recharge which were supposed to be performed at different moments after the drawdowns occurred in the aquifers by means of QSI3D and introduce the obtained results to the consolidation model to compute the rebounds of the aquitards.

The hydrogeological model is shown in Table 1. A pumping scheme was assumed with the following distribution, i.e., 34 % (22,000 m^3/d) from Nakhon Luang aquifer, NL; 25% (16,000 m^3/d) from Phra Pradaeng aquifer, PD; 25% (16,000 m^3/d) from Nonthaburi aquifer, NB; 8% (5,000 m^3/d) from Samkhok aquifer, SK; each 4% (2500 m^3/d) from Phayathai aquifer, PT and 4% (2500 m^3/d) from the Bangkok aquifer, comprising the Upper and Lower Bangkok aquifers, i.e., UBK and LBK. The drawdowns in each aquifer were calculated from a steady-state analysis by means of the QSI3D program as 2.18, 8.14, 31.16, 46.7, 33.2, 8.69, 1.25 and 0.5 m for UBK & LBK, PD, NL, NB, SK, PT, TB, PN aquifers, respectively. They were introduced together with the additional data such as the vertical stress, the maximum past pressure, the consolidation coefficient (Cv), the consolidation ratio (CR) and the recompression ratio (RR) to the consolidation model. The consolidation coefficient (Cv) was assumed to be 5 m^2/y, the recompression ratio (RR) was assumed to be 20% of the compression ratio (CR), the overconsolidation ratio (OCR) was assumed 1.1 for the clay layers. The soil compressions and rebounds due to discharge and recharge were calculated for six clay layers, i.e., Bangkok Clay (BC), Lower Bangkok Clay (LBK), Phra Pradaeng Clay (PD), Nakhon Luang Clay (NL), Nonthaburi Clay (NB) and Samkhok Clay (SK). The results are shown in Figure 6a-f. The black dot curves show how the compressions of the clay layers are increased with time since a certain drawdown occurred in the aquifers. The empty dot curves show the effects of artificial recharge, which was supposed to be done after 5, 10, 15, 20, 25, 30, 35, 40, 45 years after the drawdown occurred in the aquifer. The recharge have brought some rebounds in the clay layers, recovering a part of the compression caused by discharge. The final compressions can be read on the ordinate axis by projecting horizontally from these empty dot curves. From the curves in Figures 6a-f it can be noted that only for the Upper Bangkok and Samkhok aquifers where head drops were small, the induced pore pressure changes in the BC and SK clay layers were not enough to increase the effective stress over the maximum past pressure of the soil, recharging could recover almost totally the compressions. For the other clay layers recharging can recover some part up to 30 - 40% of the compressions caused by the previous pumping. Say, in the case of PD clay, due to a head drop of 8 m (80 kPa) in the overlying aquifer (the Lower Bangkok) and a head drop of 36 m (360 kPa) in the underlying aquifer (Phra Pradaeng), the cumulative compression got a value of 3 cm after 1 year and 7.5 cm after 15 years (Figure 6c). A recharge which started after 5 years could bring back the final settlement from 7.5 cm to 4.7 cm (37%), a recharge starting after 10 years could bring it back from 7.5 cm to 5.4 cm (28%). Recharges after 10, 15 or 20 years in this case gave the same results. Thus, once a head drop occurred in an aquifer a sooner recharge is better.

4. CONCLUSIONS

A FEM quasi-3D model was developed and applied for modelling of groundwater flow in the Bangkok aquifer system. The model proved to be a useful tool in modelling of responses to artificial recharge in the Bangkok system when discharge and recharge would take place in different aquifers at different patterns of pumping. In coupling with a consolidation model, the rebounds of the injected soil medium due to recharge could be calculated. The results of a recharge analysis depend on the magnitudes of the drawdowns occurred in the components of the aquifer system, on the geotechnical properties of the soil column, especially, the vertical stress, the maximum past pressure, the consolidation coefficient (Cv), the consolidation ratio (CR) and the recompression ratio (RR). In the actual analysis, recharge could bring back about 30% to 40% of the total compression of the clay layers when the maximum past pressure of the soil was overcome by the effective stress. This percentage, although may change depending on the input soil parameters, can be seen as an upper limit of the recharge-induced rebound for deep clay layers in the Bangkok aquifer system.

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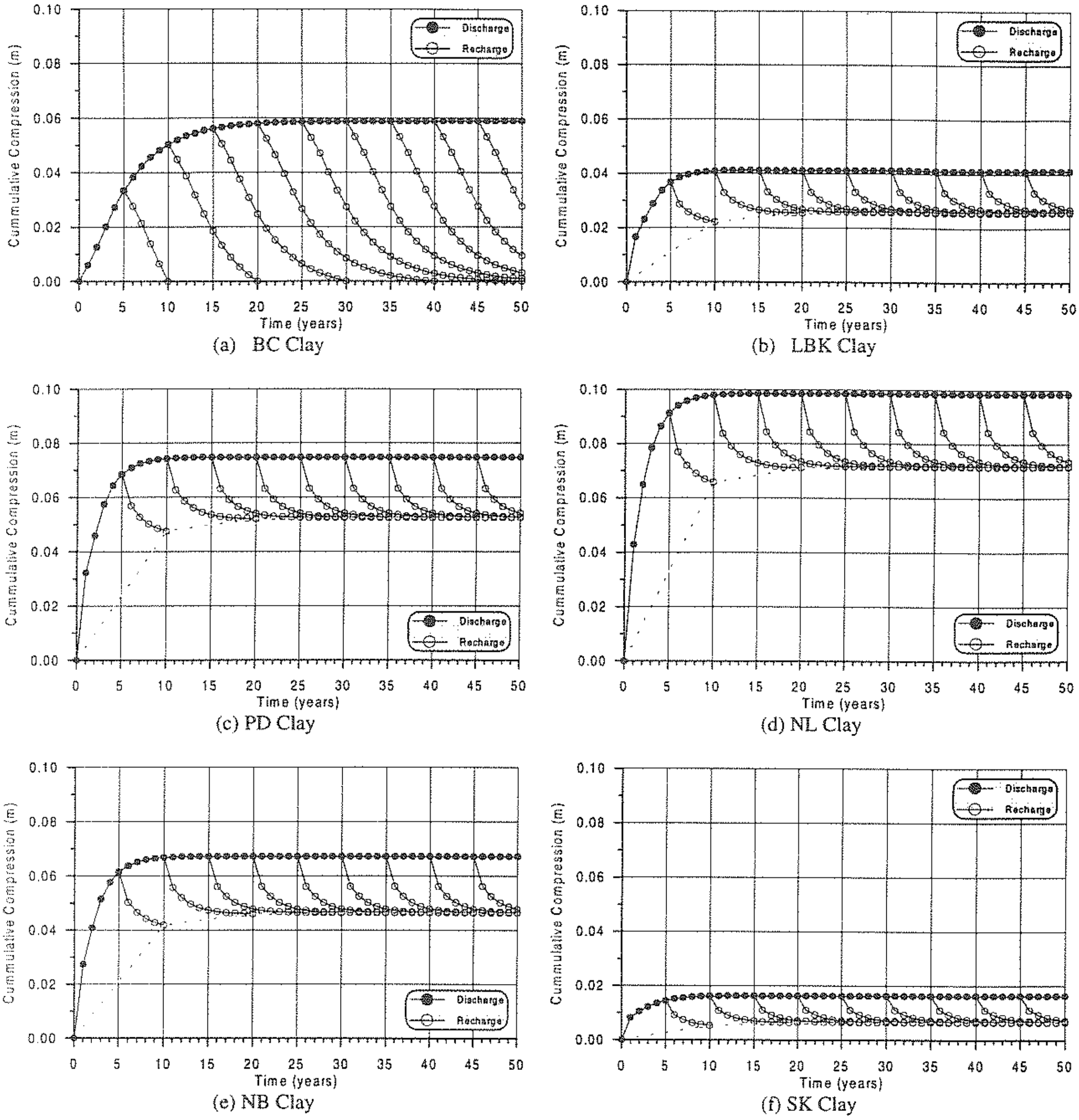

Figure 6 Cumulative compression and rebound of the clay layers in the Bangkok aquifer system due to discharge and recharge. Calculations with $Cv = 5\ m^2/y$, $CR/RR = 0.2$.

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